**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans.

We have a normal distribution with = 45 and = 8.0.

Let X is the amount of time it takes to complete the repair on a customer's car.

To finish in one hour you must have X ≤ 50 .so,the question is to find Pr(X > 50)= 1 - Pr(X ≤ 50). Z = (X -)/ = (X - 45)/8.0 Thus the question can be answered by using the normal table to find Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4% Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or **0.2676.**

Hence, Answer is Second Option which is **0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans.**

**A.**

We have a normal distribution with = 38 and = 6.

Let X be the number of employees. So according to question Probabilty of employees greater than age of 44= Pr(X>44) Pr(X > 44) = 1 - Pr(X ≤ 44). Z = (X - )/ = (X - 38)/6 Thus the question can be answered by using the normal table to find Pr(X ≤ 44) = Pr(Z ≤ (44 - 38)/6) = Pr(Z ≤ 1)=84.1345% Probabilty that the employee will be greater than age of 44 = 100-84.1345=15.86%

Therefore, the statement that “More employees at the processing center are older than 44 than between 38 and 44” is **TRUE.**

**B.** Probability of employees less than age of 30= Pr(X<30).

Z = (X -)/ = (30 -38)/6

Thus, the question can be answered by using the normal table to find

Pr(X ≤ 30) = Pr(Z ≤ (30 -38)/6) = Pr(Z ≤ -1.333)=9.12%

So, the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48(or 36 employees).

Therefore the statement B of the question is also **TRUE.**

1. If *X1* ~ *N*(μ1, σ12) and *X*2 ~ *N*(μ2, σ22) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans. As we know that,if** *X1* ~ *N*(μ1, σ12) and *X*2 ~ *N*(μ2, σ22) are two independent variables then X1 + X2 ~ N(μ1+ μ2, σ12 + σ22) and X1-X2 ~ N(μ1- μ2, σ12 + σ22)

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Similarly if Z = aX1 + bX2, where X1 and X2 are as defined above, i.e Z is linear combination of X1 and X2, then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

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| 2X1~ N(2 u,4 σ^2) and |
| X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 ) |
| 2X1-(X1+X2) = N( 4µ,6 σ^2) |

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans.**

Given:  p(a<x<b) = 0.99 ,mean =100,standardDeviation = 20

From the above details, we have to exclude area of .005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values

**Using Python**

Z value is given as stats.norm.ppf (pvalue)

Z value at 0.5th percentile is given as

                                         Z (0.5) = stats.norm.ppf (0.005)= -2.576

Z value at 99.5 percentile is given as

                         Z (99.5) = stats.norm.ppf (0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

      a = - (20\*2.576) + 100= **48.5**

      b = (20\*2.576) +100= **151.5**

Two values symmetric about mean for the given standard normal distribution are [**48.5, 151.5**]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans.**

1. 95% of the **probability lies**between 1.96 **standard deviations**of the **mean.**

Thus **range**is = (12-1.96\*5, 12+1.96\*5)

= ($2.2 M, $22.8M)

= (Rs.99M, Rs.1026M)

1. Trying to solve but, it won’t solved.
2. Trying to solve but, it won’t solved.